$$\begin{array}{ccc} & \overbrace{\quad\quad\quad}^{\text{Heads }(R_{II})} & \\ \text{Red (High income) } (R_I) & \left\{\begin{matrix} + & o \\ + & o \end{matrix}\right\} & \text{Black (Lower income) } (B_I) \\ & \underbrace{\quad\quad\quad}_{\text{Tails }(B_{II})} & \end{array}$$

A

vaguely equivalent $\sim$

$$\begin{matrix} o & + \\ o & + \end{matrix}$$

$$\wedge$$

$$\begin{matrix} + & + \\ o & o \end{matrix}$$

definitely equivalent $\sim$

$$\wedge$$

$$\begin{matrix} o & o \\ + & + \end{matrix}$$

contradiction

B

$$\begin{matrix} o & o \\ + & o \end{matrix} \qquad \begin{matrix} o & + \\ o & o \end{matrix}$$

$$\wedge \qquad\qquad \wedge$$

$$\begin{matrix} o & + \\ o & o \end{matrix} \qquad \begin{matrix} o & o \\ + & o \end{matrix}$$

Damascus race

Is income of local Cadillac dealer higher than mine?

	Wait	Strike (p)
Wait	U_{11}, V_{11}	U_{12}, V_{21}
Strike (q)	U_{21}, V_{12}	

p, q known within intervals.

US Goals: a) raise U_{11} (but may lower V_{11}, hence $\hat{q}$, hence raise p)
→ peace, cold war, limited war, accidents, budget

b) raise U_{12} (but may raise V_{21}, hence lower $\hat{q}$, hence raise p)
(or: may raise U_{21}, ~~...~~ enough so that $\frac{U_{21}'}{U_{12}'} > \frac{U_{21}}{U_{12}}$, so that $\hat{p}$ is lowered, so that p is raised; may also raise q → raising p.)

civil damage
and military outcome

c) raise U_{21} (see above; though mitigated if U_{12} is also raised).
to ~~lower~~ raise U_{11}

d) lower p (but may lower U_{11}) q, hence lower

raise: $V(\text{Wait}) - V(\text{Strike}) = (V_{11} - V_{21}) - q^*(V_{11} - V_{12})$

SU critical risk $\hat{q}$: $\dfrac{V_{11} - V_{21}}{V_{11} - V_{12}}$

US critical risk $\hat{p}$: $\dfrac{U_{11} - U_{21}}{U_{11} - U_{12}}$

---

1) alternative postures:

	$0, 0$	$-100, -20$
	$-20, -100$	

"Unstable"

$\hat{q} = \frac{1}{5}$

$\hat{p} = \frac{1}{5}$

$V(\text{Wait}) - V(\text{Strike}) = (V_{11} - V_{21}) - q^*(V_{11} - V_{12})$

if $q^* = 0$, this $= 20$

$0, 0$	$-100, -80$
$-80, -100$	

$\hat{q} = \frac{4}{5}$

$\hat{p} = \frac{4}{5}$

$V(\text{Wait}) - V(\text{Strike}) = 80$ if $q = 0$

"Stable" (but effect on U_{11}?)

SHIFT FROM (a) to (b) MAY REPRESENT HIGHER BUDGET: "ARMS RACE"
LIKEWISE FROM (b) TO (a) MAY REPRESENT "DISARMAMENT."

(2) Strategic equivalence:

0, 0	−100, −40
−80, −100	

either: a) SU AICBM vs. Polaris

CD based on evacuation + blast shelters

or b) US shift to soft missiles; or Polaris becoming vulnerable.

or US C+C vulnerable; or US warning degraded.

(3) Civil Defense: (to say that given change in US payoffs will "provoke" SU attack, or raise p, is to say contingent: ~~(i) q is sensitive to change~~ 1) ~~total~~ effect of change on US payoffs, hence on $\hat{p}$; 2) sensitivity of q to change in $\hat{p}$; (3) sensitivity of p to change in q $\hat{p}$; 4) SU payoffs: $\hat{q}$.

0, 0	−100, −80	or "objectively"	0, 0	−150, −100
−80, −100			−100, −150	

Change to: a) evac + blast + fallout objectively: 0, 0 −40

−20, −100

payoffs 0, −60

−40

$\hat{p}$ ~~same~~ lower (somewhat)

b) fall-out: objectively 0, 0 −40

−40 − −60

payoffs 0 −60

−60 − −80

$\hat{p}$ higher

Note: vN-M utilities: difference between 40 − 150 million dead may not be "worth" a war"; i.e. $U(-40) \geq (1-p)\,U(0) + p\cdot U(-150)$ only if $p \geq .6$ or higher.

BUT IT IS NOT "SMALL"

CD cont:

If $\hat{p}$ is lowered, this will raise q: but how much?
and how sensitive is SU decision: how low is $\hat{p}$?
e.g., how high is v_{21}, compared to v_{12}?

(4)

100 MT weapon.

invulnerable	0, 0	-80, -80
used in 1st strike	-60, -100	
hidden, not used in 1st strike	0, 0	-60, -80
	-80, -100	

(5) Central war tactics (go "within" v_{12} and v_{21})

Both spasm war: (see (1))
Both control 1st strike, spasm second:

0, 0 -

Both control: 0, 0
-60, -

⑥ Berlin: do we *want* q *even if* u_{11} is lowered drastically?

A) US monopoly: $0, 0 \qquad -15, -100 \qquad \hat{q} = 1$

$-10, -100 \qquad \hat{p} = \frac{2}{3}$

US Type I high: $V(\text{Wait}) - V(\text{Strike}) = (100) - q(0) = 100$

But US Type II also high: $U(W) - U(S) = (10) - p(15)$

SU deterrence was "unreliable"; unstable to shifts *down* in u_{11} or *up* in p.

Then SU acquired capability to hit US bases in Europe + NATO allies

B) $0, 0 \qquad -30, -80 \qquad \hat{q} = \frac{8}{9}$

$-15\text{-}20, -90 \qquad \hat{p} = \frac{1}{2} - \frac{2}{3}$

US Type I down slightly

US Type II up or down (we had new *reason* to strike)

C) SU ability to hit US (small, vulnerable)

$0, 0 \qquad -60, -70 \qquad \hat{q} = \frac{6}{7} - 1$

$-40, -60\text{-}90 \qquad \hat{p} = \frac{2}{3}$

US Type I down somewhat, but still high

SU Type I up greatly: not so much in $\hat{p}$ but in sensitivity to drops in u_{11}

i.e. US Type II down sharply; though not "vanished," for *big* drops in u_{11} combined with moderately high p.

[illegible]

		Cold war	Limited war conventional
D)	Wait	0, 0	-30, 20
US	Strike	-40, ~~-60-70~~	-40, -60
NATO	Wait	0,0	-60, 20
	US doesn't Strike	~	-75, 15
	US does Strike		-90, -60

Worst NATO fear if SU threatens/gestures to go to limited war

a) SU might not believe US Strike, so would move.

b) US would Strike, if any fighting developed.

Hence, NATO preferred policy:

a) make counter-threat of US Strike as credible as possible (~~lower~~ raise U_{21}) and as frightening (lower V_{12}; generally they prefer this; ~~to~~ want to scare SU without encouraging US)

b) Make sure no fighting develops; negotiate, esp. if risk of fighting looms.

Raising U_{21} will increase US Type II, in short-run; but it will increase effectiveness of SU threats, if NATO doesn't believe that SU "really" believes US will GO; but NATO does believe.

(e.g. US controlled war talk may convince NATO US means to GO; but they fear it won't convince SU.)

Long-run:	0, 0	-60, 20
Conventional		-10, -20

I

Ignorance & Decision

1. Given various circumstances, what to do. Specifically: when you don't know what will happen when you take a particular action.

"Don't know" is vague; I wish to be more precisely. But ~~what I want to talk about more~~ vagueness is precisely what I want to talk about; and, if I can, I want to talk about it precisely.

How is it reasonable to act when the consequences of your acts are not merely uncertain but are extremely vague? What does this mean? How can we identify, ~~to~~ measure, express vagueness; and what influence does it have on decision. Suppose we can say, meaningfully, that some actions are much more ambiguous than others; Is this a difference that makes a difference?

2. A complete theory of action under uncertainty exists: Ramsey.

Man behaves — when he stops to think — "as if" he obeyed Bernoulli principle; assigned numbers, ...

3. Moreover, since this is normative, proponents recommend that "you" not only behave "as if" you did this, but that you do just that;

R & S.

4. What if, when you ask yourself your opinions, you get no answer? Or you get several answers, and when you ask how to compare them, you get, "I don't know";

E.g. you're the President (IKE), and the USIB splits?

R+S: Force yourself; define family of dists (e.g., all "diffuse") take "most acceptable" one (any these that are not contradicted by definite opinions); any one of them "expresses" definite opinions (w. Jeffreys); but "diffuse" one has property of "letting sample decide" — which "it will do anyway" if it's big enough.

5. S, dF, R+S, G point out it may not make any difference which prior dist you use. But sometimes it will. Nevertheless, you must act "as if" you had definite opinions — they give rules, questions to ask yourself — that will generate a dist. even when your mind is vague.

Why? Because otherwise you would violate axioms; which, they conjecture — you wouldn't want to do if you stopped to think.

6. Suppose you did obey Bernoulli principle; if we knew one variable we could measure the other. Bayes, vN-M. Ramsey: assume neither, derive both, starting with special choices: 0, 1 payoffs; if we assume Bernoulli principle.

1	0	1	1	0	1	1	1	0	0
0	1	1	0	0	0	0	1	1	0

$E \quad F$

	$E \cap \bar{F}$	$F \cap \bar{E}$	$E \cap F$	$\bar{E} \cap \bar{F}$
I	1	0	1	0
II	0	1	1	0

$I > II \Rightarrow E > F$

Build up "body of choices" like this, get an inferred "body of beliefs."

Will it be "consistent" with axioms of qual prob?

Will it be true that: $E > F,\ F > G \Rightarrow E > G$

$E > F \Leftrightarrow \bar{E} < \bar{F}$?

Suppose

1	0	0
0	1	0
1	0	1
0	1	1

$E > F$, but $\bar{E} > \bar{F}$! But this is ruled out by P2.

Likewise: $10 0 0 → $100 0 P4

0 $10 0 → 0 $100

1	0	0
0	1	0
0	0	1

$E > F,\ F > G,$

~~Ex~~

~~the empirical~~

	E	$\bar{E}$		
I	20	0	10	−10
II	10	10	0	0

If we take regrets: I′ 0 10

II′ 10 0

we can't tell difference between I + II: i.e. difference is that II offers a definite outcome, I offers two possible outcomes, uncertain.

Assignment of utilities indicates (only) that this doesn't make a difference when $pr(E) = \frac{1}{2}$ (more generally: $\exists\ E \ni$ I i II). But that doesn't mean the difference is irrelevant to behavior for all E: ~~though axioms imply~~ ~~that~~ criterion (as does minimax regret). This seems most psychologically unsound in just those situations (of high ignorance) when minimax regret is proposed.

Axioms imply that:

I	10	0	−10	or I	a	c
II	0	0	0	II	b	b
III	−10	0	10	III	c	a

I i III i II for some E $\Rightarrow$ $pr(E) = \frac{1}{2}$ and $u(b) = \frac{u(a) - u(c)}{2}$

so if $u(a) = 10$, $u(c) = -10$, $u(b) = 0$

and $\Rightarrow$ for all E, not (II > I and II > III)

i.e. "the difference" between II and (I, III) can never "make a difference such that" both II > I and II > III; i.e. in any pair

$(\mathrm{I}, X) = (\mathrm{II}, X) = (\mathrm{III}, X)$ for <u>all</u> X and all E.
(unless $\mathrm{II} = X$)

involving II, <u>either</u> I could be substituted for II without affecting ordering, <u>or</u> III could be. or <u>both</u>. (i.e., if $\mathrm{II} > X$, then either $\mathrm{I} > \mathrm{II} > X$ or $\mathrm{III} > \mathrm{II} > X$ or $\mathrm{I} = \mathrm{III} = \mathrm{II} > X$.
If $\mathrm{II} < X$, either $\mathrm{I} < \mathrm{II} < X$ or $\mathrm{III} < \mathrm{II} < X$, or $\mathrm{I} = \mathrm{II} = \mathrm{III} < X$.

BUT suppose that for some E, $\mathrm{II} > \mathrm{I}$ and $\mathrm{II} > \mathrm{III}$.
Then it might be that $\mathrm{II} > X > \mathrm{I}$ and $\mathrm{II} > X > \mathrm{III}$, ~~so~~ ~~then~~ In this case, difference would "make a difference"; <u>neither</u> member of the set (I, III) would be an "<u>acceptable substitute</u>" for II.

Coherence:

<u>If</u> you max math expect with weights, ~~sum~~ weights must not add to > 1; or else

suppose $\sum p_i = k > 1$ $\qquad = 1 < k$

	p_1	p_2		
I	1	1	$v(\mathrm{I}) = 1$	$\frac{k+1}{k}$
II	$\frac{1}{k}$	$\frac{1}{k}$	$v(\mathrm{II}) = \frac{1}{k}(\sum p_i) = 1$	

But ~~suppose~~ $\sum p_i < 1$ $\qquad \sum p = k < 1$

1	1	$= 1$
$\frac{1}{k}$	$\frac{1}{k}$	$= 1$

Let I : II : III for some E.

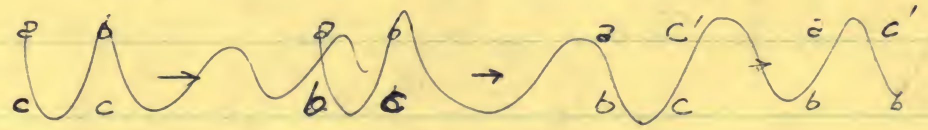
Then for any E, I : II $\overset{\text{IV}}{\Leftrightarrow}$ I : II : $\underline{\text{III : IV}}$

I	a	b
II	b	a
III	c	c
IV	c′	c′

Assume $a > c > b$ and $a > c' > b$

Suppose $c' > c$. Then IV > III

Then I > III and II > III



1) a b and 3) b a

2) c c 4) c c

a b → a b → b a → b a → c′ a

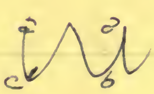
c c   b c   b c   b b   c′ b

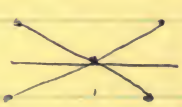
a b →

c b

↘ c′ c′ → c′ c′

b c





a b   a b

b a

c′ c′

c c

$$\begin{pmatrix} a & b & b & b \\ b & a & b & b \\ c & c & b & b \end{pmatrix}$$

and

$$\begin{pmatrix} b & b & a & b \\ b & b & b & a \\ b & b & c' & c' \end{pmatrix}$$

and

$$\begin{matrix} c & c & b & b \\ b & b & c & c \end{matrix}$$

then

$$\begin{pmatrix} b & b & c & c \\ b & b & c' & c' \end{pmatrix}$$

and

$$\begin{pmatrix} c & c & c & c \\ c' & c' & c' & c' \end{pmatrix}$$

P3

Suppose

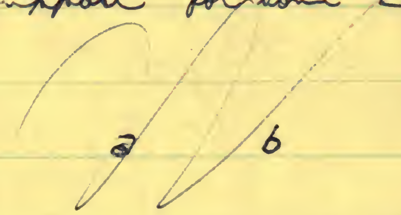
$$\begin{array}{cccc} b & b & c & c \\ b & b & c' & c' \\ \hline b & b & c' & c' \\ b & b & b & a \\ \hline a & b & c' & c' \\ a & b & b & a \end{array}$$

$$\begin{array}{cccc} c & c & b & b \\ b & b & c' & c' \\ \hline c & c & b & b \\ b & b & b & a \\ \hline a & b & b & b \\ b & b & b & a \\ \hline a & b & a & b \\ b & b & a & a \\ \hline a & b & a & b \\ a & a & b & b \\ \hline b & b & a & b \\ b & a & b & b \\ \hline b & b & c' & c' \\ c & c & b & b \\ \hline b & b & c' & c' \\ b & b & c & c \end{array}$$

$$\begin{matrix} b & c & b & b \\ b & c & b & a \end{matrix}$$

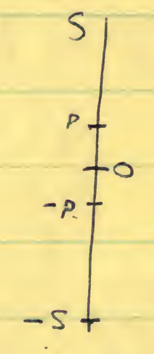
$$\begin{array}{cccc} a & a & b & b \\ b & a & b & a \\ \hline b & b & a & a \\ b & a & b & a \\ \hline b & b & a & b \\ b & a & b & b \\ \hline b & b & c' & c' \\ c & c & b & b \end{array}$$

Suppose for some E:



a	$-a$	a	b
$-a$	0 $\rightarrow$	b	a
		c	c

a	b
0	0
(a)	(c)
S	0
(d)	
p	p
(b)	
$-S$	0
(d')	
$-p$	$-p$



Suppose $\begin{pmatrix} a & b \\ b & a \\ c & c \end{pmatrix}$

and $\begin{pmatrix} d & d' \\ c & c \\ d' & d \end{pmatrix}$

and if $\begin{pmatrix} a & c \\ d & d \end{pmatrix}$

then $\begin{pmatrix} b & c \\ d' & d' \end{pmatrix}$

Proof: Suppose $\begin{matrix} d' & d' \\ b & c \end{matrix}$ $\quad \begin{matrix} d' & d \\ b & c \end{matrix}$ $\quad$ or Suppose $\begin{matrix} b & c \\ d' & d' \end{matrix}$

$\begin{matrix} c & c \\ b & c \\ b & a \\ b & c \end{matrix}$ $\quad \begin{matrix} c & a \\ c & c \\ c & a \\ a & b \end{matrix}$

$\begin{matrix} d' & d' \\ b & b \\ d' & d' \\ d & d' \end{matrix}$

contr contradicts Pa

Suppose

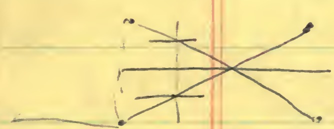
a b b b
b a b b
c c b b
b b c c
b b a b
b b b a
~~b b c c~~
b b a a
a a b b

Implied by #4

then <u>not</u>
b b c c
b b a b
<u>and</u>
b b c c
b b b a

Proof: Suppose both of above: ~~Then~~ Suppose
b b a b
b b c' c'

and
b b b a
b b c'' c''



then
b b c c
b b c' c'

and
b b c c
b b c'' c''

i.e. $c > c'$
$c > c''$

a b b b
b b c' c'

and
b a b b
b b c' c'

a b b b
c' c' b b

and
b a b b
c' c' b b

a a b b
b b a a
then ~~b b~~

~~Also~~

	$\bar{B}$ A		$\bar{A}$ B	
f_A	c c		b b	
f_B	b b		c c	
g_A	a a		b b	
g_B	b b		a a	

c b | d
b c | d

and $\begin{pmatrix} b & b & c & c \\ b & b & b & a \end{pmatrix}$

Proof: Suppose

$$\begin{array}{cccc} b & b & c & c \\ b & b & a & b \\ \hline b & b & c & c \\ b & b & a & \\ \hline a & b & c & c \\ a & b & a & b \\ \hline b & a & c & c \\ a & b & a & b \\ \hline c & c & c & c \\ c & c & a & b \\ \hline b & b & a & a \\ c & c & a & b \\ \hline b & b & b & a \\ c & c & b & b \\ \hline b & b & b & a \\ b & b & c & c \end{array}$$

Either $\begin{matrix} b & a & b & b \\ c & c & b & b \end{matrix}$

and $\begin{pmatrix} a & b & b & b \\ c & c & b & b \end{pmatrix}$

(assume

P1

(assume $\begin{pmatrix} c & c & c & c \\ a & a & b & b \\ b & b & a & a \end{pmatrix}$

contradiction

$$\begin{array}{cccc} b & b & c & c \\ b & b & a & b \\ \hline \end{array}$$

$$\begin{array}{cccc} b & b & b & a \\ b & b & c & c \\ \hline b & b & b & a \\ b & b & b & a \\ \hline a & a & b & b \\ c & c & a & b \\ \hline a & a & b & a \\ c & c & a & a \\ \hline a & a & b & a \\ a & a & c & c \\ \hline b & b & b & a \\ b & b & c & c \end{array}$$

cont.

cont.

THEOREMS

TH 1 By P4:

$$\begin{array}{c|cccc} & \multicolumn{2}{c}{\overbrace{\qquad\qquad}^{A}} & \multicolumn{2}{c}{\overbrace{\qquad\qquad}^{B}} \\ \text{I} & a & a & b & b \\ \text{II} & b & b & a & a \\ \\ \text{III} & c & c & b & b \\ \text{IV} & b & b & c & c \end{array} \Bigg) \text{ then}$$

$a > c > b$

$$\text{I} = \text{II} \iff \text{III} = \text{IV}$$

see Savage, p. 31

TH 2 Proof: I, II, III, IV ; A, B ; a, b, c, b ; are such that

1. $b < a$, $b < c$

(2a) $\text{I} = a$, $\text{III} = c$ for $s \in A$

$\text{I} = b$, $\text{III} = b$ for $s \in \sim A$

(2b) $\text{II} = a$, $\text{IV} = c$ for $s \in B$

$\text{II} = b$, $\text{IV} = b$ for $s \in \sim B$

3. $\text{I} = \text{II}$

Then by P4: $\text{III} = \text{IV}$

TH. 2:

$$\text{if}\ \left(\begin{array}{lcccc} \text{I} & a & b & b & b \\ \text{II} & b & a & b & b \\ \text{III} & c & c & b & b \\ \text{IV} & b & b & c & c \end{array}\right) \text{ and } \left(\begin{array}{lcccc} \text{V} & b & b & c' & c' \\ \text{VI} & b & b & a & b \\ \text{VII} & b & b & b & a \end{array}\right) \text{ then}$$

Th: if $\text{I} = \text{II} = \text{III} = \text{IV}$, and $\text{V} = \text{VI} = \text{VII}$; then $\text{I} = \text{II} = \text{III} = \text{IV} = \text{V} = \text{VI} = \text{VII}$

and $c = c'$

Proof (1): ~~IV = V~~ (a) $\text{III} = \text{IV} \Rightarrow \text{VIII} = \text{IX}$, where

$$\begin{array}{lcccc} \text{VIII} & a & a & b & b \\ \text{IX} & b & b & a & a \end{array}$$

by Th. 1.

(b) $\text{I} = \text{II}$, $\text{VI} = \text{VII}$, $\text{VIII} = \text{IX} \Longrightarrow \text{I} = \text{II} = \text{VI} = \text{VII}$

by proof in my article, Th. 0.

c) Hence $\text{IV} = \text{V}$ $(= \text{I} = \text{II} = \text{III} = \text{VI} = \text{VII})$,

by P1

Proof (2) Suppose

$$\begin{array}{lcccc} \text{IV} & b & b & c & c \\ \text{V} & b & b & c' & c' \end{array} \qquad \text{IV} > \text{V}$$

$$\begin{array}{cccc} c & c & b & b \\ b & b & c' & c' \end{array} \quad \text{ON}$$

$$\begin{array}{cccc} a & b & b & b \\ b & b & b & a \end{array} \quad \text{P1}$$

$$\begin{array}{cccc} a & b & a & b \\ b & b & a & a \end{array} \quad \text{P2}$$

$$\begin{array}{cccc} a & b & a & b \\ a & a & b & b \end{array} \quad \text{P1}$$ (Given $\text{III} = \text{IV}$ and Theorem 1)

$$\begin{array}{cccc} b & b & a & b \\ b & a & b & b \end{array} \quad \text{P2}$$

$$\begin{array}{lcccc} \text{V} & b & b & c' & c' \\ \text{IV} & b & b & c & c \end{array} \quad \text{P1}$$

CONTRADICTION

~~Theorem 2.~~

Theorem 3.

$$\begin{array}{c|cccc}
\mathrm{I} & a & b & b & b \\
\mathrm{II} & b & a & b & b \\
\mathrm{III} & c & c & b & b \\
\mathrm{IV} & b & b & c & c \\
\hline
\mathrm{V} & b & b & a & b \\
\mathrm{VI} & b & b & b & a
\end{array}$$

Th: If $\mathrm{I} = \mathrm{II} = \mathrm{III} = \mathrm{IV}$

then <u>not</u> both $\mathrm{IV} > \mathrm{V}$ and $\mathrm{IV} > \mathrm{VI}$

(i.e. either $\mathrm{IV} \leq \mathrm{V}$ or $\mathrm{IV} \leq \mathrm{VI}$, or both, in which case $\mathrm{IV} = \mathrm{V} = \mathrm{VI}$)

(or: either $\mathrm{V} > \mathrm{IV} > \mathrm{VI}$ or $\mathrm{VI} > \mathrm{IV} > \mathrm{V}$ or $\mathrm{V} = \mathrm{IV} = \mathrm{VI}$)

Proof: Suppose $\mathrm{IV} > \mathrm{V}$ and $\mathrm{IV} > \mathrm{VI}$.

(1)

$$\begin{array}{c|cccc|l}
\mathrm{IV} & b & b & c & c & \\
\mathrm{V} & b & b & a & b & \\
\hline
 & a & a & c & c & \text{P2} \\
 & a & a & a & b & \\
\hline
 & c & c & a & a & \text{P1 (and Th. 1)} \\
 & a & a & a & b & \\
\hline
 & b & a & a & a & \text{P2 (and P1)} \\
 & a & a & a & b & \\
\hline
 & b & b & b & a & \text{P2} \\
 & a & b & b & b & \\
\hline
\mathrm{VI} & b & b & b & a & \text{P1} \\
\mathrm{IV} & b & b & c & c &
\end{array}$$

CONTRADICTION

(2)

$$\begin{array}{c|cccc|l}
\mathrm{IV} & b & b & c & c & \\
\mathrm{V} & b & b & a & b & \\
\hline
 & c & c & c & c & \text{P2} \\
 & c & c & a & b & \\
\hline
 & b & b & a & a & \text{P1 (and by Theorem 4)} \\
 & c & c & a & b & \\
\hline
 & b & b & b & a & \text{P2} \\
 & c & c & b & b & \\
\hline
\mathrm{VI} & b & b & b & a & \text{P1} \\
\mathrm{IV} & b & b & c & c &
\end{array}$$

Stet

CONTRADICTION

Theorem (4)

$$\text{if} \quad \left(\begin{array}{lcccc} \text{I} & a & b & b & b \\ \text{II} & b & a & b & b \\ \text{III} & c & c & b & b \end{array}\right.$$

$$\text{and} \quad \left(\begin{array}{lcccc} \text{IV} & a & a & b & b \\ \text{V} & b & b & a & a \\ \text{VI} & c & c & c & c \end{array}\right) \quad \text{then}$$

Th: if $\text{I} = \text{II} = \text{III}$ and $\text{IV} = \text{V}$, then $\text{IV} = \text{V} = \text{VI}$

Proof.

$$a)\quad \left.\begin{array}{lcccc} \text{VII} & a & b & c & c \\ \text{VIII} & b & a & c & c \\ \text{VI} & c & c & c & c \end{array}\right)$$

$\text{VI} = \text{VII} = \text{VIII}$ by P2

$$b)\quad \begin{array}{lcccc} \text{I} & a & b & b & b \\ \text{III} & c & c & b & b \\ \hline & a & b & b & b \\ & b & b & c & c \\ \hline \end{array} \quad \text{Th. 1}$$

$$\begin{array}{lcccc} \text{IV} & a & a & b & b \\ \text{VIII} & b & a & c & c \\ \hline \end{array} \quad \text{P2}$$

$$\begin{array}{lcccc} \text{IV} & a & a & b & b \\ \text{VI} & c & c & c & c \\ \hline\hline \end{array} \quad \text{P1} \qquad \text{Q.E.D.}$$

Thus: 'How much should I pay for a bet: $\begin{array}{cc} E & \tilde{E} \\ 10 & 0 \end{array}$?

Suppose E is: highly ambiguous

Answer (Savage): a) Are you indifferent between $\begin{array}{cc} E & \tilde{E} \\ 10 & 0 \\ 0 & 10 \end{array}$?

b) If so: pay up to amount you would pay for

$\begin{array}{cc} H & T \\ 10 & 0 \end{array}$ '

If "bet" is $\begin{array}{cc} E & \tilde{E} \\ -10 & 0 \end{array}$; ask no more than if bet were

$\begin{array}{cc} H & T \\ -10 & 0 \end{array}$.

Suppose agent looks into urn I and tells you:

either A (proportion of Red is between $0 - \frac{1}{3}$)

or B (" " " " $\frac{1}{3} - \frac{2}{3}$)

or C (" " " " $\frac{2}{3} - 1$) .

Suppose he says: "B".

Raiffa: U_I U_{II} 50:50

① $i \begin{array}{c} aRb \\ bRa \end{array}$ ② $i \begin{array}{c} aRb \\ bRa \end{array}$

③ $aR_I b$ i $aR_{II} b$? If not, contradiction

(Need not even offer $bR_I a$ vs $bR_{II} a$)

Flexibility: (see Marschak – Nelson)

Let a_1 determine the set of actions (A_2) at t_2.

a) actions at A_2 should be characterized by objective outcomes; because one reason for flexibility is that payoff function (strategic objectives) may change between t_1 and t_2.

a_1 more flexible than $a_2 \Leftrightarrow A_1 \supset A_2$.

b)

Suppose payoff fn. will stay same from t_1 to t_2. Then actions characterized by payoffs.

It may be that payoffs to "same" actions are lower if chosen out of A_1 than out of A_2. Then A_1 doesn't "include" A_2, if we characterize actions by payoffs.

But it may be possible to represent payoffs to actions as $a_2^j \equiv (x^j) - (y_{A_2^i})$, where $(y_{A_2^i})$ is a vector depending on the set A_2^i (determined by a_1^i) and independent of the action a_2^j; and (x^j) is a vector depending on the action a_2^j and independent of the set A_2^i.

In particular, assume (y^i) is a constant vector. Then it represents the "cost of choosing from A_2^i."

$(y^i)-(y^k)$ = opportunity cost (regret) of choosing from A_2^i rather than A_2^k. (assume $y^i > y^k$).

Special case: $y^i \geq y^k \Leftarrow A^i \supset A^k$ (defined objectively).

Say a_1^i is more flexible than a_1^k if $A_2^i \supset A_2^k$ when actions are characterized by the payoff vectors (x^j).

~~Special case: (y^i) = 0 for all i.~~

Special case: assume $y^i = 0$ for all i such that A_2^i consists of one action only (no choice possible at t_2; only choice is a_1 at t_1).

Value of flex. is not related to the amount of info I expect to gain between t_1 and t_2 but the value of info I "expect" (conservatively?).

Hyp: How much I will pay for flex. is related to ambiguity of my expectations on value of info to be gained.

		E	$\bar{E}$
A_2^i	I	1	0
	II	0	1

A_2^k: 1 0 $\quad$ A_2^m: 0 1

value of $A_2^i = A_2^k$. then,

Person for whom [pr $(E) = \frac{1}{2}$] : value of $A_2^i = A_2^k = A_2^m = \frac{1}{2}$

he will pay premium of 0 for A_2^i (the "more flexible" set).

But for minimaxer, for whom $0 \le$ prob $(E) \le 1$; $A_2^k = A_2^m$; but value of $A_2^i = \frac{1}{2} > A_2^k = A_2^m = 0$.

He will pay premium of $\frac{1}{2}$ for A_2^i.

Thus, if $a_1^i \to A_2^i$, $a_1^k \to A_2^k$, $a_1^m \to A_2^m$;

he will prefer a_1^i to a_1^k, and will pay premium up to $\frac{1}{2}$, even though (if it a_1^m had been offered) he would have been indifferent between a_1^k and a_1^m; and would have been indifferent between a_1^i and flipping a coin between a_1^k and a_1^m.

Straight minimax regret notion more like "hedging."

When the effect of opponent's strategy on my payoffs is only way to know "what he did," the hedging strategy sacrifices chance of gaining info during sequential process.

("You can't see the cards unless you pay." (Suppose you don't "see" them even then, but umpire announces whether you were beaten or not. Kriegspiel.)

Flexibility: Define physical outcome— consequences "state of the world," relevant aspects — in terms of n dimensions. Specify relevant m contingencies, states of the world that affect outcomes.

An action maps ~~a~~ states of the world into consequences.

A set of actions is larger than another set with respect to a given contingency (state), ~~&~~ and with respect to n dimensions of output, if for that contingency, the set of different values of the n dimensions "available" within the set is larger by some measure; in particular, if it includes the other set.

Since the costs of different actions may be different under different sets, define "availability" w.r.t. a fixed budget at the time of choice. Thus, set A could be more flexible than set B at a high budget, but less flexible at a low budget.

Value of flexibility will depend on: 1) exact nature of uncertainty about the state of the world; 2) various sorts of uncertainty expected to prevail at t_2, time of choice. 3) possible variation in payoff function; 4) ~~action~~ payoffs to less flexible alternative actions or sets of actions.

Choice of an action at time t rules out further choice, further control; it includes choice of a decision rule for acting on further info

H	T
a	b
c	c

Prove that for any $E \ni$

	E	$\tilde{E}$
I	a	b
II	b	a
III	c′	c′

$I = II = III, \quad c' = c$

Consider either

H	E T	$\tilde{E}$
a	b	b
b	a	b
c	c	b
a	a	b
b	b	a′
c′ H	c′	c′ T

or

E	$\tilde{E}$	
a	b	b
b	a	b
c′	c′	b
a	a	b
b	b	a
c	c	c

a_1 Back A [illegible]

a_2 Back B

e_0 experiment

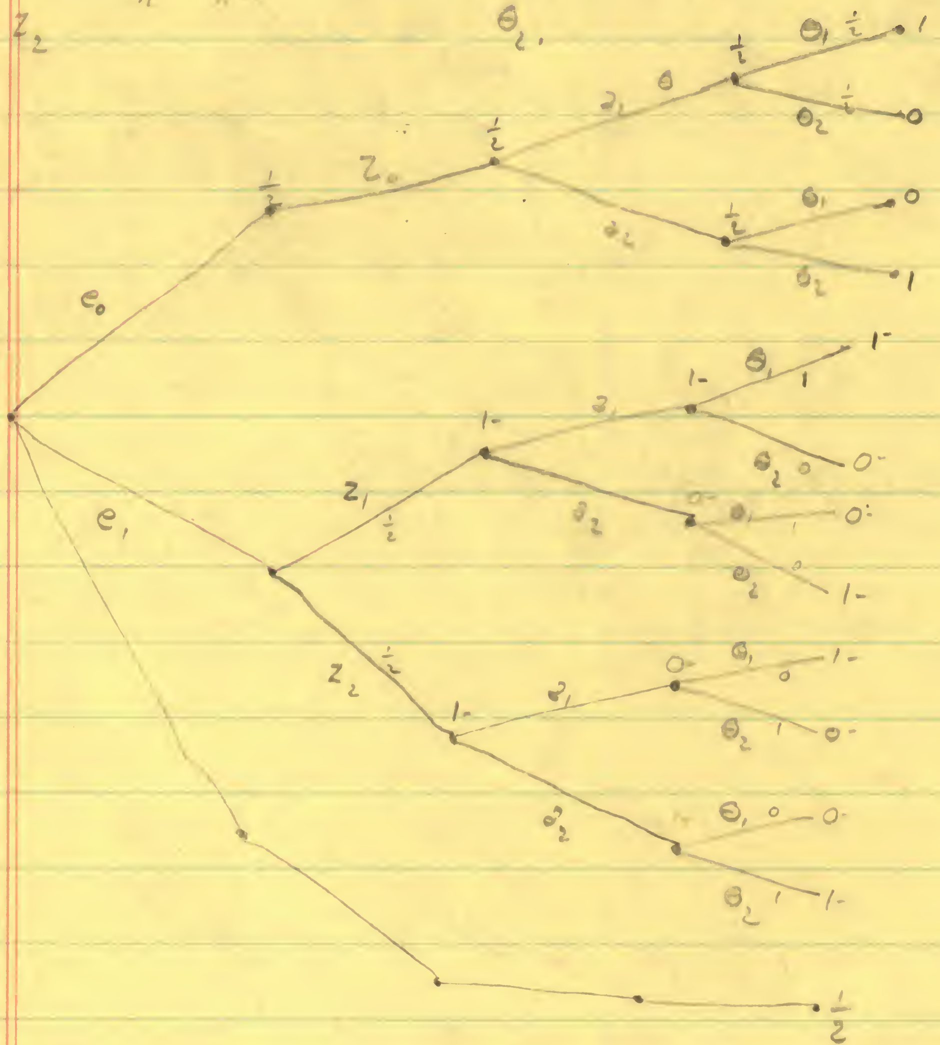
e_1 Back A + B through to prototype

θ_1 A is cheaper

θ_2 B is cheaper

z_1 outcome of e_1 favorable to θ_1

z_2 " " " " θ_2.



Flexibility:

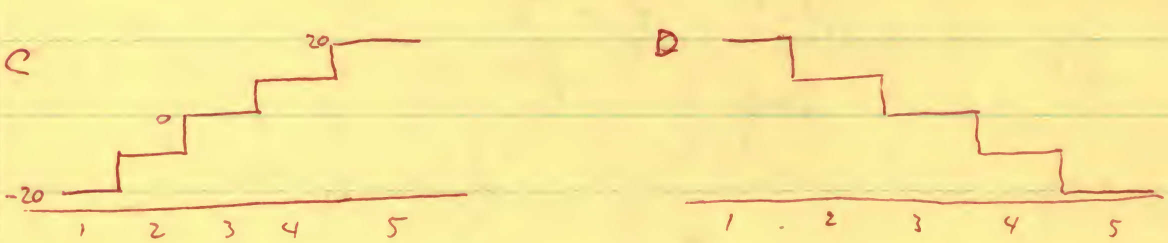
If we assume payoff function fixed, then: a) can define flexibility in terms of different payoffs; so set of actions "available" under given contingency will be smaller (some physical actions will have same payoff); (b) can count only undominated actions in set, w.r.t. whole set of contingencies, or w.r.t. some subset of contingencies (e.g. 2 events; or even 1, if values are defined only as a partial ordering) in evaluating a set of actions.

Thus, if a "large" set of actions A has a single dominant strategy a', $v(A) = v(A')$, when $A' = a'$ (less flexible).

1) When does $A \supset B \Rightarrow v(A) > v(B)$?

2) How to measure "value of flexibility": $v(A) - v(a')$, where $a' \in A$, $v(a') \geq v(a'')$ for all $a'' \in A$.

(w.r.t. "least flexible subset" of A).



(1-5 are dial settings; A-D are events, or different payoffs functions; graphs are payoffs).

	A	B	C	D
1	-10	10	-20	20
2	0	0	-10	10
3	10	-10	0	0
4	0	0	10	-10
5	-10	10	20	-20

20	0	40	0	[illegible]
10	10	30	10	15
0	20	20	20	15
10	10	10	30	15
20	0	0	40	[illegible]

Marginal Value of flexibility:

Compute minimum expected regret of ~~...~~ preserving some subset of $(n-1)$ actions, vs. set of n actions.

1-5:	10	10	20	20					
1-4	10	10	10	20	0	0	10	0	2.5
2-5	10	10	20	10	0	0	0	10	2.5
1-3	10	10	0	20	0	0	20	0	5
3-5	10	10	20	0	0	0	0	20	5
2-4	10	0	10	10	0	10	10	10	7.5
1+5	-10	10	20	20	20	0	0	0	5
2+3	10	0	0	10	0	10	20	10	10
4+5	0	10	20	-10	10	0	0	30	10

2-4 has higher average regret than (1,5).

If $A \supset B$, you must do at least as well with A as with B for any contingency. ~~Must have~~ $v(B) \leq v(A)$. Expected regret for B w.r.t. A $\geq$ ~~expected~~ 0. Expected regret for B w.r.t. A. $= 0$.

But if, say, two actions within the set had, between them, the best payoff for every contingency, there would be no value in adding more actions. Never a value in adding an action unless for some contingency it is best (assuming you will know event with certainty before acting).

Flexibility is a way of lowering expected regret. If a single action were available with same regrets as 1-4, it would be just as good. You can't tell just by looking at two sets of payoffs corresponding to two strategies, which is more "flexible." (esp. if cost of flex. has been subtracted).

See Marschak + Nelson, p. 9

	1	2	3	4	5	6
I	0	1	0	1	0	1
II	1	0	1	0	1	0

Assume rect. dist. Value of $I \cdot v(II) = v(I, II)_{\text{no info}} = \frac{1}{2}$.

Value of info that event is (1,2), (3,4) or (5,6) = 0

Value of info that event is (odd) or (even) = $\frac{1}{2}$.

Value of flexibility when you expect perfect info = value of perfect info = expected regret with less flexible set compared to more flexible set, when you expect perfect info later.

Value = f(info, flex).

Value of info : (1,2,3), (4,5,6) = $\frac{1}{6}$

$\dot{t}_1$ t_2

⇒

Suppose I start knowing: (1,2,3)

$v(I, II) = v(II) = \frac{2}{3}$

Value of info: (1,2) or (2,3) or (1,3)

$\frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 1 = \frac{2}{3}$. Value of info = 0

✓

But if I start with (1,2,3,4). $v = \frac{1}{2}$

and learn (1,2,3), (2,3,4), (1,2,4) or (1,3,4): $v = \frac{2}{3}$

Value of info = $\frac{1}{6}$

Or: start with (1–6), go to (1,2), (3,4), (5,6): value of info or flex = 0 [go to (1,3), (2,4), (5,6): value of info = $\frac{1}{3}$]

But start with (1–4), go to (odd) or (even): value of info = $\frac{1}{2}$.

For flexibility to have any "value", it is necessary (but not sufficient) that either (a) expectations may change, as a result of new info or further analysis; or (b) payoffs may change, as a result of new info, or analysis, or "learning," etc.

(~~Either~~ For either of these, it is necessary but not sufficient that initial expectations and/or payoffs be uncertain to be regarded as possible).

Whether given flexibility will appear valuable, and to what degree, will depend on; e.g.: 1) the precise way in which it is believed that uncertainty will be reduced or changed (e.g. new info, of greater reliability than initial info, may simply ~~contradict~~ cast doubt on earlier info: canals on Mars, cross-examination of witnesses, general strategy of defense (of a client who doesn't have clear-cut proof of innocence).

2) the payoffs to the various inflexible alternatives, and to the choices permitted by the flexible strategy; e.g. the costs of flexibility.

3) The lead-time of ~~the~~ info of given credibility and the speed of response (BMEWS).

4) The required credibility for given responses; its likelihood, its lead time, response time.

Flexibility is a form of insurance (not vice versa) premised on the possibility of acquiring valuable information; its value is equivalent to the expected (or index) value of information for the flexible set of actions as opposed to the expected value of some less flexible set. Other forms of insurance can be evaluated on the assumption that info does not change or improve (e.g. that it worsens) or that true choice will not be made; i.e. that initial choice will "really" determine action.

Minimax regret could be interpreted as:

Way of evaluating a) flexibility, assuming perfect info at t_2;
or b) perfect info, given flexible set at t_2,
when no restriction can be put on "reasonable" probs., and
$\alpha = 0$ $(\rho \cong 0)$.

~~If flexible action~~ If "cost" of flexibility is independent of actual event which obtains, flexible ~~action~~ set of actions will have constant regret w.r.t. less flexible set (This does not guarantee minimax regret unless cost is 0; ~~doesn't~~).

Like evaluating value of info, with given flexibility; with "fixed" cost of info.

Suppose there is a possibility that uncertainty will not decrease, & even a small prob. that it will increase (that signals may occur increasing uncertainty). Flexibility per se won't "insure against" this possibility. "Insurance" actions "look acceptable" against initial or higher uncertainty. Include them in flexible set. AJW stresses.

Decision to drop 2 bombs on Japan, before test. (Decision could have been sequential, flexible, but wasn't).

Difficulty of really "postponing decision" — "leaving alternatives open". Flanders offensives. "Cost" of switching from "publicly expected" alternative or of "delaying decision" may be such as really to drop out certain alternatives from fixed-budget comparison.

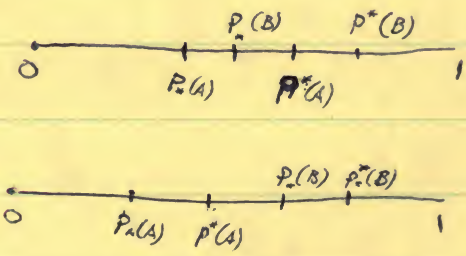
A	B	$\bar{A} \cap \bar{B}$						
a	b	b	$= p_*(A)$	$=$	1	0	0	$\frac{1}{3}$
b	a	b	$= p_*(B)$		0	1	0	$\frac{1}{6}$
a	b	a	$= q_*(B)$		1	0	1	$\frac{1}{2}$
b	a	b	$= q_*(A)$		0	1	1	$\frac{2}{3}$

$$p_*(A) \leq [1 - q_*(A)] = p^*(A)$$
$$p_*(B) \leq [1 - q_*(A)] = p^*(B)$$

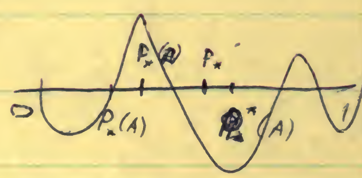
$P2:$ $p_*(A) > p_*(B) \Leftrightarrow q_*(B) > q_*(A)$

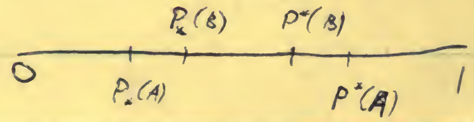
$p_*(A) = p_*(B) \Leftrightarrow q_*(B) = q_*(A)$

$p_*(A) < p_*(B) \Leftrightarrow q_*(B) < q_*(A)$



Rules out





$0 \quad 1 \quad 0 = 1 - q_*(B)$

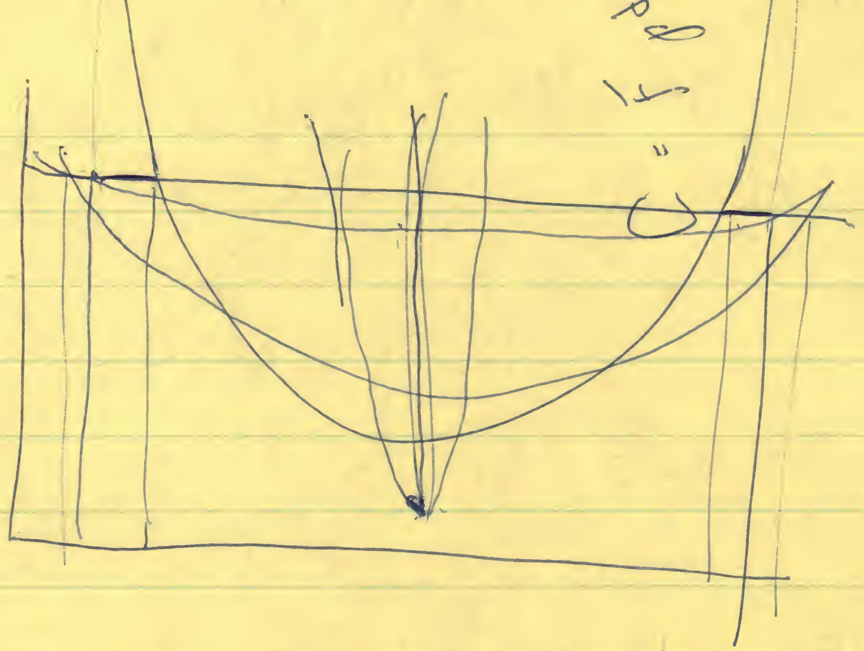
$1 \quad 0 \quad 0 = 1 - q_*(A)$

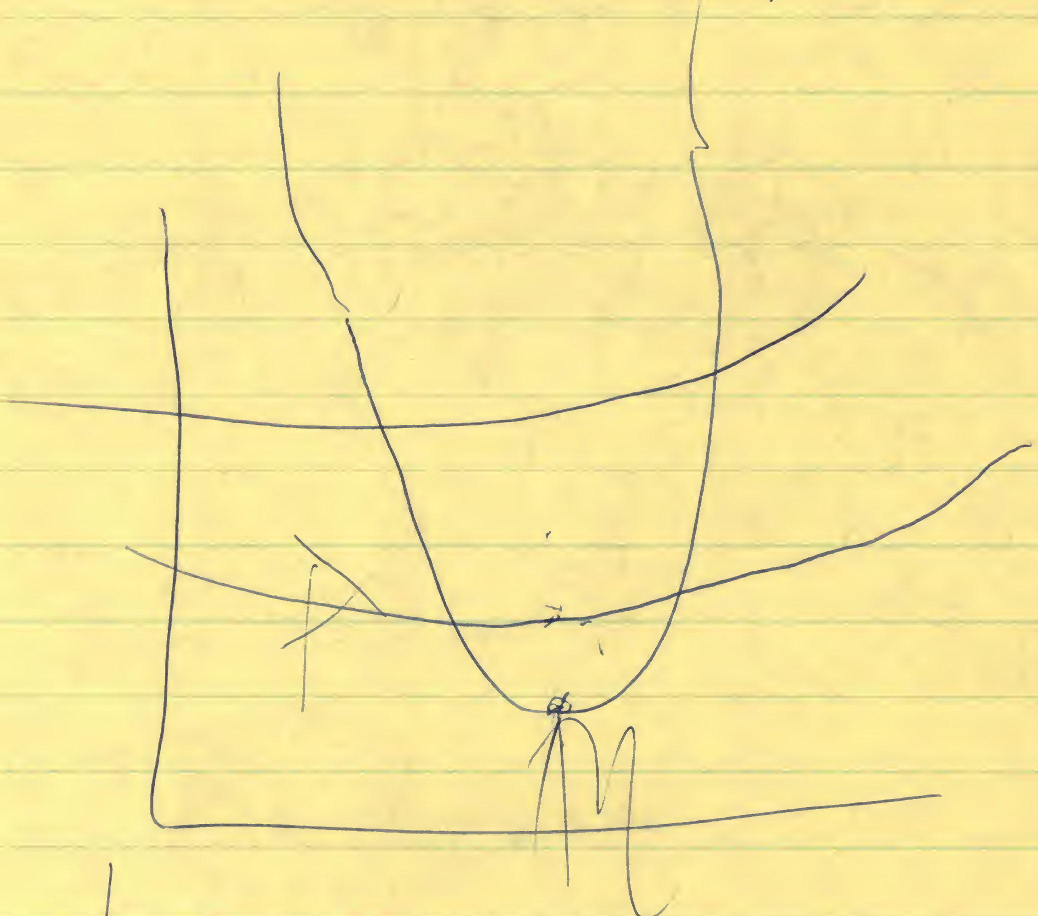
$(1 \quad 0 \quad 1) \leq (1 \quad 1 \quad 1) - (0 \quad 1 \quad 0) = (1 \quad 0 \quad 1)$

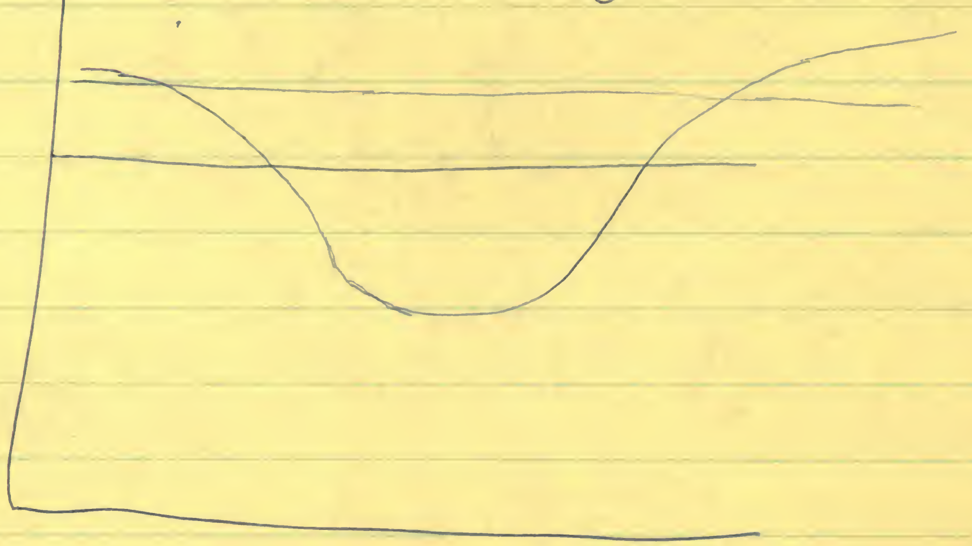
$v(1 \quad 0 \quad 1) = \frac{1}{2} < 1 - v(0 \quad 1 \quad 0) = 1 - \frac{1}{6} = \frac{5}{6}$

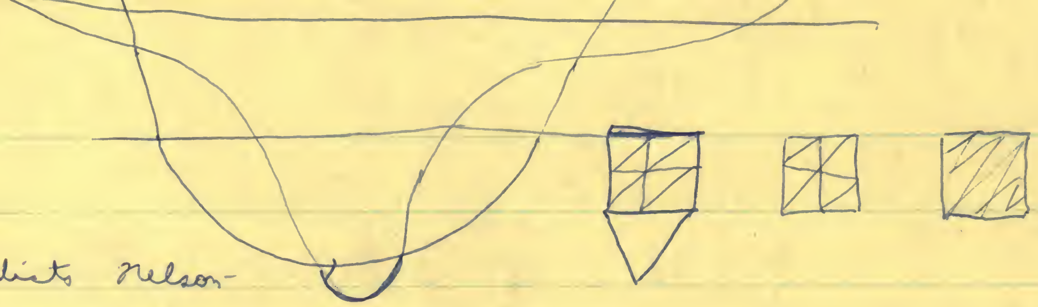
$0 \quad 1 \quad 0$

$p_*(B) \quad p_* \quad p_*$

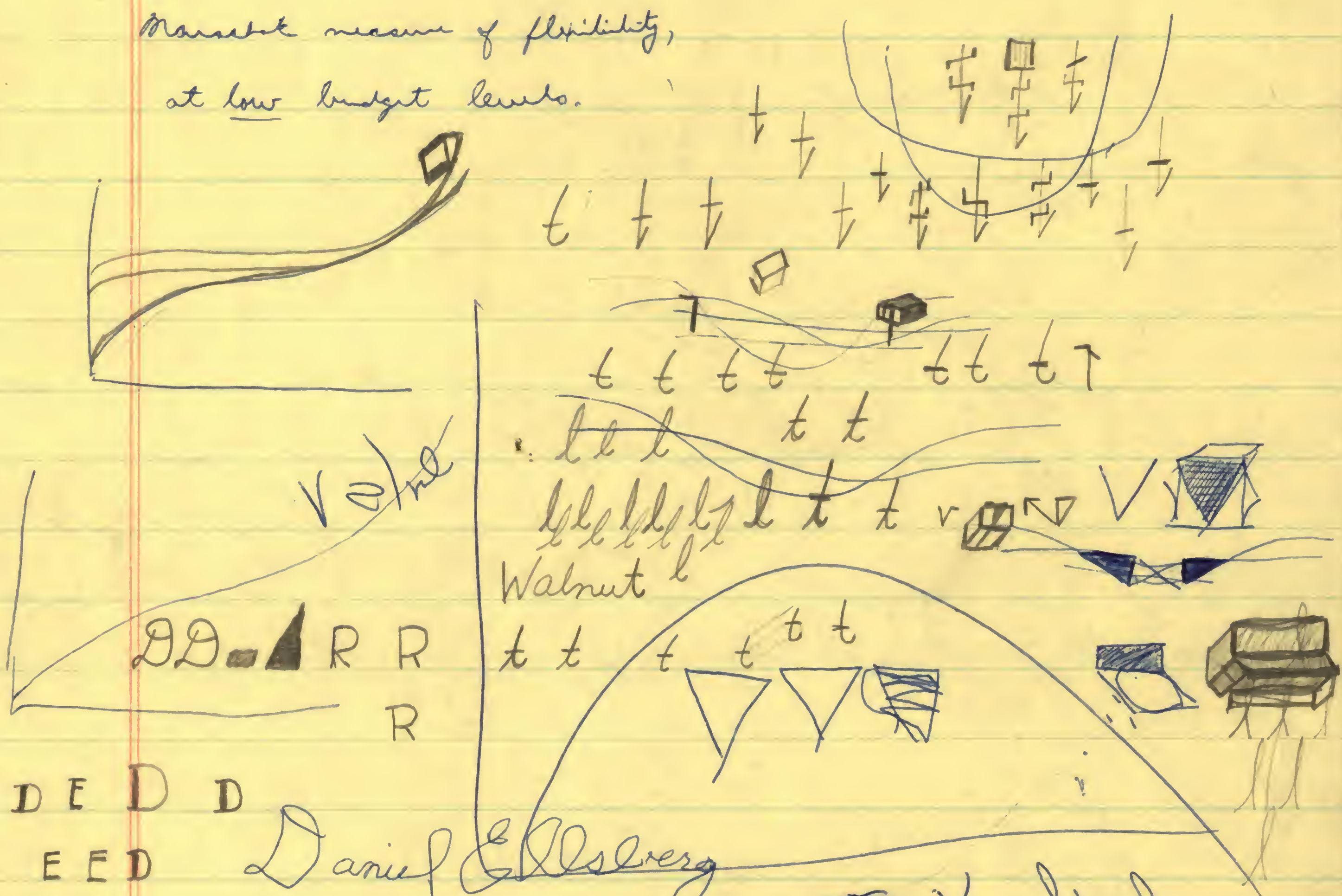








Contradicts Nelson-Marschak measure of flexibility, at low budget levels.



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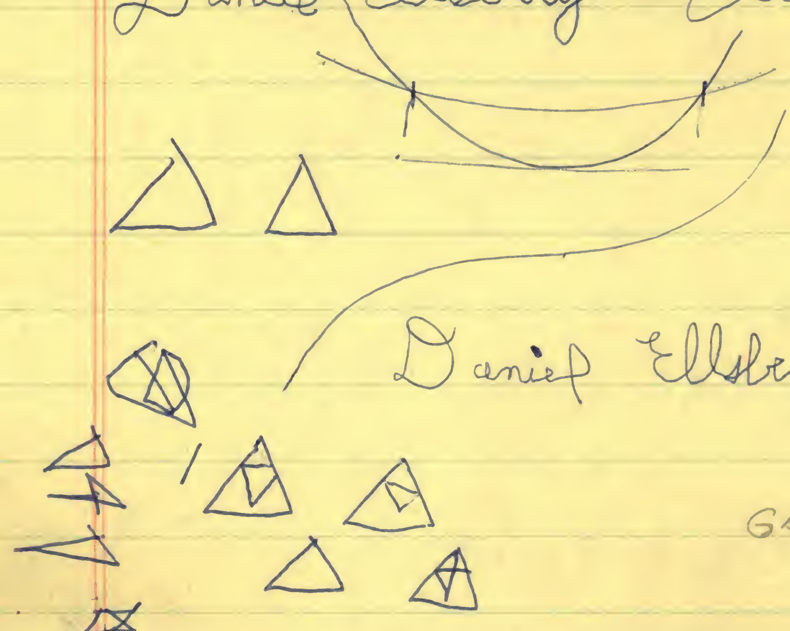

$\frac{1}{3}$	1	0	0	$\frac{1}{3}$	$\frac{1}{3}$
	0	1	0	$\frac{1}{3}$	$\frac{1}{6}$
	1	0	$\emptyset$	$\frac{2}{3}$	$\frac{1}{2}$
	0	1	1	$\frac{2}{3}$	$\frac{2}{3}$

Daniel Ellsberg

GR-30971 X9979

$1 \quad 0 \quad 0 \quad \tfrac{2}{3} \qquad -1 \quad 0 \quad 0 \quad -\tfrac{1}{3} \quad 0 \quad 1 \quad 1$

" $\qquad\qquad\qquad\qquad$ "

$0 \quad 1 \quad 0 \quad \tfrac{1}{6} \qquad 0 \quad -1 \quad 0 \quad -\tfrac{1}{2} \quad 1 \quad 0 \quad 1$

$1 \quad 0 \quad 0 \qquad\qquad 0 \quad 1 \quad \cancel{0}$

$p_*(A) \quad p_*(A) \quad p_*(A) \qquad q_*(A) \quad q_*(A) \quad q_*(A)$

$1 \quad 0 \quad 1 \qquad\qquad 0 \quad 1 \quad 0$

$q_*(B) \quad q_*(B) \quad q_*(B) \qquad p_*(B) \quad p_*(B) \quad p_*(B)$

Columns: ~~$p^*(A) \leq q^*(A)$~~ $p_*(A) \leq 1 - q_*(A)$

$v(1\ 0\ 0) \leq 1 - v(0\ 1\ 1)$

(Savage wants to prove these equal)

$-1 \quad 0 \quad 0$

$-p^* \quad -p^* \quad -p^* \qquad v(-1\ 0\ 0) = v(0\ 1\ 1) - 1 =$

so $\quad -p^* = q_*(A) - 1$

or $\quad p^* = 1 - q_*$

To assume $p_* = p^*$ is to assume $p_* = 1 - q_*$

and to assume $v(1\ 0\ 0) = 1 - v(0\ 1\ 1)$

[or to assume $v(1\ 0\ 0) = v[(1\ 1\ 1) - (0\ 1\ 1)]$]

$1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0$

$0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0$

$v(1\ 0\ 0) = v(0\ 1\ 0) \Leftrightarrow v(1\ 0\ 1) = v(0\ 1\ 1)$

$v(1\ 0\ 0) - v(0\ 1\ 0) = v(0\ 0\ 0) \Leftrightarrow v(1\ 0\ 1) - v(0\ 1\ 1) = 0$

$(1\ {-1}\ 0) = 0 \qquad\qquad (1\ {-1}\ 0) = 0$

JR 5 ?

	E	$\bar{E}$		
I	1	0	0	0
II	p	p	0	0
III	0	1	0	0
IV	q	q	0	0
V	0	0	q	q
VI	0	0	0	1
VII	0	0	p'	p'
VIII	0	0	1	0

Then $p' = p$, $\text{II} = \text{VII} = \text{I} = \text{VIII}$

(i.e. given that $p = 1 - q$ for some E, and given p', q' for event F, with $p = p'$, then $p' = 1 - q'$, $q' = q$

By ~~P3~~ P2, if $p_* = 1 - q_* = p^*$ for some E, this must hold for all E' such that $p_*' = p_*$; because by P4, ~~if~~ this implies $q_*' = q_*$; so if $p_* = 1 - q_*$, then $p_*' = 1 - q_*'$

(But what guarantees that this ever holds? P5?)

1	0	0	p_*
0	1	0	p_*'
1	0	1	q_*'
0	1	1	q_*

$\begin{cases} \text{II} > \text{I} & p_*' > p_* \\ \text{III} > \text{IV} & q_*' > q_* \end{cases}$ $\qquad p^{*\prime} < p^*$

$\begin{cases} \text{I} > \text{II} & p_* > p_*' \\ \text{IV} > \text{III} & q_* > q_*' \text{ or } (1 - q_*) = p^* < p^{*\prime} \end{cases}$

P2 gives complete ordering, but doesn't guarantee that $p_* = p^*$. It does, however, if there exists an event with "definite" p $=1-q$ corresponding to every event $\ni p_* = p$. P2 rules out one "interval" enclosing another.

$$\rho \cdot y_0 + 1-\rho \left[\alpha \cdot (y_{max} - y_0) + (1-\alpha)(y_{min} - y_0) \right]$$

$I = \frac{1}{3}$ $\quad$ $II =$

Raiffers $\underline{II}-\underline{I}$, $\underline{IV}-\underline{III}$

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LA. 34, Cal.

Thus, <u>person</u> <u>calling</u> committee has no interest in ~~committee~~ <u>their</u> using minimax regret or in <u>his</u> using it to resolve their reported disagreement.

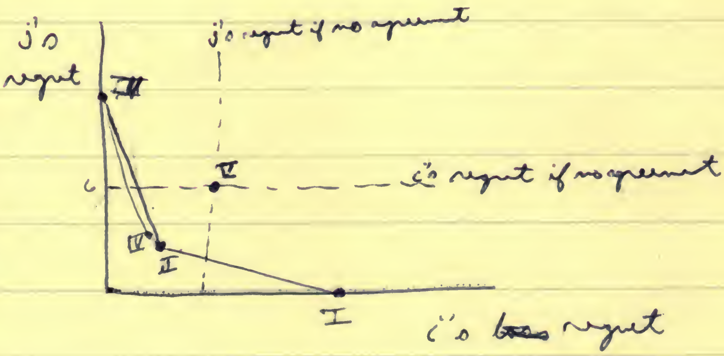
<u>Hyp</u>: Where minimax regret looks good to an <u>individual</u>, it is in circumstances when it looks good on Ellsberg index; and in such cases, (~~i.e.~~ hence, in <u>all</u> cases) some <u>other</u> action may look better. (may have higher max regret, <u>but</u> .... higher minimum, or higher max, or higher $\bar{x}_{est}$).

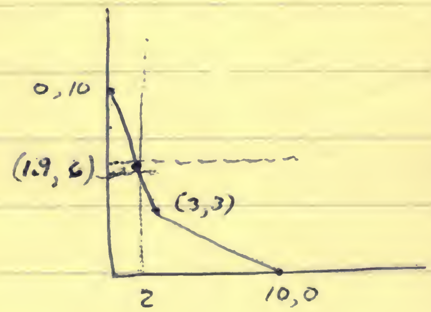
For group,

A peculiarity of this bargaining situation is that acts are available — involving observations — with very low regret.

Regrets happen to express relevant payoffs in certain problems. But <u>strict minimax</u> is no more generally valid with regrets than with losses.

		i	j
	I	+10	0
	II	3	3
	III	0	10
	IV	2	3.5
Don't agree	V	5	6





One who "cares less" if no agreement is reached is ~~in position~~ in "better bargaining position" to influence outcome within the acceptable set (What if he makes <u>threats</u> to fail to agree?)

	β_1	β_2
I	0	-14
II	-3	-13
III	-1	-20
IV	-4	-10

If I <u>believe</u> $p(\beta_1) = 1$ (not sure), Expected loss to III = 1. But <u>max</u> loss to III = 10, whereas <u>max</u> loss to II is 3 (which is also expected loss).

see Savage, p. 159.

On the other hand, Expected loss to I is 0, with max loss of 4: looks better than II (the max loss is not <u>enough</u> worse to outweigh)

The game:

I	10	0	-10
II	0	0	0
III	-10	0	10

Person who preferred either mixed strats $(1,0,0)$ or $(0,0,1)$ to $(0,1,0)$ or to $\left(\frac{1-\lambda}{2}, \lambda, \frac{1-\lambda}{2}\right)$, $0 \le \lambda \le 1$, will

a) be disobeying Savage axioms (as is minimaxer; but for different reasons)

b) be following Ellsberg rule with $\alpha > \frac{1}{2}$ (more weight to "good" ambiguous possibilities than to bad)

c) also prefer any non-symmetrical mixed strategy to any number of $\left(\frac{1-\lambda}{2}, \lambda, \frac{1-\lambda}{2}\right)$.

		2P, 1D	2D, 1P	1D, 2P	1P, 2D
Guess	D	0 + 20	−8 + 10	20	− 18
Guess	P	−8 + 20	0 + 10	12	−10

30 $\quad$ 60 either all B, or 30R, 30B

$R \qquad \overbrace{R \qquad B}$

R $\quad$ B $\quad$ R or B

30 $\quad$ 30 $\quad$ 30

30 $\quad$ 60 either all B or 30R, 30B

R	R	B	
0	0	−8	0, −8; $\frac{1}{3}$ − $\frac{2}{3}$
−8	0	0	0, −8; $\frac{1}{3}$ − $\frac{2}{3}$

0	−8	−8	0, −8; $\frac{1}{3}$
−8	−8	0	0, −8; $\frac{1}{3}$ − $\frac{2}{3}$

0	−8	1	1	or	0	0
−8	0	−7	9		−8	8

Not equivalent

3	3
−5	11

Savage

p. 183

1. $\max_g \tilde{L}(f, g) = \max_i L(f; i)$

$$\begin{matrix} 0 & -10 \\ -5 & 0 \end{matrix} \quad \rightarrow \quad \begin{matrix} 0 & 10 \\ 5 & 0 \end{matrix}$$

		Loss		Regret	
10	-3	-10	<u>3</u>	1	<u>16</u>
-9	13	<u>9</u>	-13	<u>20</u>	0
11	12	<u>-11</u>	-12	0	<u>1</u>

?

Conjecture: Regret is <u>relevant</u> only when one <u>knows</u> "the truth" (either certainty or certain prob. dist) ~~or~~ (in group problem) or ~~has~~ an act available which would <u>reveal</u> the truth (observation, which would lead to certainty of prob. dist).

Otherwise, the differences that <u>matter</u> are differences between a proposed action and some other action: one that someone else proposes, or that looks interesting on some other ground, or <u>would</u> be chosen "otherwise" — and even these aren't <u>solely</u> relevant when there is ambiguity.

I	10	0	0	10	0	10
II	0	10	0	0	10	10

I-II:	10	-10	0	10	-10	0
II-I	-10	10	0	-10	10	0

R + D

Don't produce	0	0	Without observation, Don't produce
~~Pr~~ Produce	-100	100	V=0

Observe, then ~~produce~~ decide:

Possible results of observation:

	$\frac{1}{2}$	$\frac{1}{2}$
	-100	100

		-100	100
i	I	1	0
	II	0	1

If -100, I won't produce; payoff 0; difference: 0

If 100, Produce; payoff 100; difference 100

V of observation: $\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 100 = 50$

Will pay up to 50.

~~A~~	-100	100
B	-100	100

Observe, then decide

~~A -100 100~~

~~Observe, then decide~~

A	B		Payoff	
-100,	-100	→	-100	$\frac{1}{4}$
-100,	100		100	$\frac{1}{4}$
100,	-100		100	$\frac{1}{4}$
100,	100		100	$\frac{1}{4}$

$V(\text{obs}) = \left(\frac{1}{4} \cdot -100 + \frac{3}{4} \cdot 100\right) - \left(\frac{1}{2} \cdot 100 + \frac{1}{2} \cdot -100\right)$

$50 - 0 = 50$

~~Observe~~ Minimaxer: ~~to~~ Payoff to observation:

$X_{est} = 50 \quad X_{min} = -100 \quad X_{max} = 100 \qquad \rho = \frac{3}{4} \quad \alpha = 0$

~~$\frac{1}{2} \cdot 50 + \frac{1}{2} \cdot -100 = -25$~~ $\frac{3}{4} \cdot 50 + \frac{1}{4} \cdot (-100) = 12.5$

~~Payof~~ Index of A: $\frac{3}{4}(50) + \frac{1}{4} \cdot (-100) = 12.5$

$-100, -100 \quad 100$

$-100, 100 \quad 100$

$100, 100 \quad -100$

$-100, -100 \rightarrow 0$

~~Value~~ Payoff of observation estimated: $75 -$

$75 - 0 = 75$

~~B~~ Index, $p = \frac{1}{4}$, $\alpha = 0$

$\frac{1}{4} \cdot 75 + \frac{3}{4} \cdot 0 = 18.75$

Index to A: $\frac{1}{4} \cdot 50 + \frac{3}{4} \cdot (-100) = -62.50$

$$\begin{array}{r} 18.75 \\ \hline 81.25 \end{array}$$

$$4\overline{)75} = 18, \quad 4,\ 35,\ 32,\ 3$$

$$\begin{array}{r} 75 \\ 12.50 \\ \hline 62.50 \end{array}$$

U_I $\qquad$ U_{II}

U_I

	R	B
I	·100	0
II	0	100

Index: $p = \frac{1}{4}$, $\alpha = 0$ = $\frac{1}{4} \cdot 50 + \frac{3}{4} \cdot 0 = 12.5$

Value of observation = $100 - 12.5 = 87.5$

U_{II}

Index: $p = 1$: 50

Value of observation: $100 - 50 = 50$

Or if observation will produce prob. dist: 0R, 2B $\quad$ 1R, 1B $\quad$ 2R, 0B

100 $\quad$ 50 $\quad$ 100

$\frac{1}{4} \cdot (75) + \frac{3}{4} \cdot 50 = 56.25$

$$\begin{array}{r} 37.5 \\ 18.75 \\ \hline 56.25 \end{array}$$

Value of obs: $56.25 - 12.5 =$

Urn I — 2 balls. 2R, 1R1B 2B

U_I

		R	B
Don't bet	I	~~x~~	~~100~~x
	II	100	0

$\rho = \frac{1}{5}$ $\alpha = 0$ $y_{est} = (\frac{1}{2}, \frac{1}{2})$

$v(U_I) = \cancel{\frac{1}{5} \cdot 50 + \frac{4}{5} \cdot 0} = \cancel{10} + \cancel{50} = 10$ (if $x < 10$)

In U_I', allow observation of balls, then choice of I or II:

$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
2R	1R1B	2B
100	50	0

$\frac{1}{5} \cdot (25 + 25) + \frac{4}{5} \cdot 0 = 10$

I	6	0	0
II	0	6	0

$\rho = 1$: $v(I, II) = 2$

With obs: $v(I, II) = \frac{1}{3} \cdot 6 + \frac{1}{3} \cdot 6 = 4$

Value of obs: $\underline{2}$

$\rho = \frac{1}{2}$ $\alpha = 0$ $v(II) = 1$, $v(I) = 2$, $v(I, II) = 2$

With observation: $v(I, II) = \frac{1}{2} \cdot 4 + \frac{1}{2} \cdot 2 = 3$

Value of observation $= \underline{1}$

III	6	0	6
IV	0	6	6

$\rho = 1$: $v(III, IV) = 4$

With obs: $v(III, IV) = 6$

Value of obs: $\underline{2}$

$\rho = \frac{1}{2}$, $\alpha = 0$. $v(III) = 2 + 1 = 3$, $v(IV) = 4$ $v(III, IV) = 4$

With obs. $v(III, IV) = 6$

Value of obs $= \underline{2}$

regret

		R	B
100	0	0	100
0	100	100	0
		50	50

Likelihoods

2R	1-1	2B
0	50	0

$\frac{1}{5} \cdot 50 + \frac{4}{5} \cdot 100 = 10$

$p = \frac{1}{5}$ $\quad \frac{1}{5} \cdot 25 + \frac{4}{5} \cdot 50 = 45$

With obs. $p = 1$ $\quad 1 \cdot 25 = 25$ expected regret

Without observation, but $p = 1$: 50

Value of obs = $\underline{25}$

Without obs, $p = \frac{1}{5}$ 90 index regret

With obs. $p = \frac{1}{5}$ 45 " "

Value of obs $\underline{45}$

Observe then decide

~~100~~	~~B~~		
100	0	100	0
30	30	50	50
0	100	0	100
100-x	100-x	100-x	100-x

$x = 70$ $\quad x = 50$

R	B		
100	0	100	0
→ 20	20		

~~x=70~~

$\frac{1}{5}\left(\frac{1}{2} 100 + \frac{1}{2} \cdot 20\right) + \frac{4}{5} \cdot 20 = 28$ vs. 20

60

If $p = 1$, 60 vs. 50

I	100	0	$y_{ext} = (\frac{1}{2}, \frac{1}{2})$
II	0	200	

$\rho = 1$: $v(I) = 50$, $v(II) = 100$, $v(I, II) = 100$

With obs: $\frac{1}{2} \cdot 100 + \frac{1}{2} \cdot 200 = 150$

Value of obs = $\underline{50}$

$\rho = \frac{1}{2}, \alpha = 0$ ~~v(I)~~ $v(I, II; \frac{1}{3}, \frac{2}{3}) = 66\frac{2}{3}$ ~~XAN~~

With obs: $\frac{1}{2} \cdot 150 + \frac{1}{2} \cdot 50 = 100$

Value of obs: $\underline{33\frac{1}{3}}$

$\rho = \frac{1}{2}$, $y_{ext} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ ~~__~~ $pr(R) = \frac{1}{3}$

$III: 2\frac{1}{3}$

$\frac{1}{2}(\frac{10}{3}) + \frac{1}{2} \cdot \frac{2}{3}$

II	0	6	0
III	6	0	6

$v(II) = 1$, $v(III) = 3$, $v(II, III; \frac{1}{2}, \frac{1}{2}) = 3$

value with obs: 6

value of obs. = $\underline{3}$

4 2 4

If $\rho = 1$: $v(II) = 2$, $v(III) = 4$, $v(III, II) = 4$

With obs: $v(III, II) = 6$

Value of obs = $\underline{2}$

6 0 0

0 6 6

3 3 3

If $pr(R)$ is also ambiguous:

$v(II) = 1$

$v(III) = \frac{1}{2} \cdot 4 + \frac{1}{2} \cdot 0 = 2$

$v(II, III; \frac{1}{2}, \frac{1}{2}) = 3$ With obs: 6

Value of obs $\underline{3}$

---

all probs ambiguous

I	6	0	6		3	3	0
II	0	6	0				

$v(I) = 1$, $v(II) = 1$, $v(II, I; \frac{1}{2}, \frac{1}{2}) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 0 = 1$

With obs: $\frac{1}{2} \cdot 4 + \frac{1}{2} \cdot 0 = 2$

Value of obs: $\underline{1}$

info would go up, because he would expect valuable info.

Moral: if outcome will still be ambiguous after observation, "conservative" will pay less for observation than Bayesian, because he is less sure of getting valuable info.

But if info will transform an ambiguous situation (all alternatives ambiguous gambles) into risk or certainty, conservative will pay more for info; value with obs. is same as for Bayesian, but value without info is less.

(Info would be worth more to conservative if he couldn't use mixed strats).

	R		
II	0	6	0
II′	0	0	6

$\rho = 1$: $v(II) = v(II') = 2$

With obs: $v(II, II') = 4$

$\rho = \frac{1}{2}$, $pr(R) = \frac{1}{3}$.

$v(II, II', \frac{1}{2}, \frac{1}{2}) = 2$

With observ.: $v(II, II') = 4$

II	0	6	0
II′	1.5	1.5	1.5

8	8	0	0
8	0	8	0
0	8	0	8
0	0	8	8

	50		~~50~~ 50 ~~25~~	
A	8	0	0	0
B	0	8	8	8

$v(A, B) = \frac{1}{2} \cdot 6 + \frac{1}{2} \cdot 4 = 5$

V with obser.: $= 8$

~~0 8 0 0~~

~~0 0 8 0~~

V of obs = $\underline{3}$ (To Bayesian, value of obs = 2)

A	0	0	0	8
B	0	0	8	0

$v(A, B) = 2$

With obs = 4

I	6	0	0
II	0	6	0

$\rho = \frac{1}{2},\ \alpha = 1$

$v(\text{I}) = 2, \quad v(\text{II}) = \frac{1}{2}\cdot 2 + \frac{1}{2}\cdot 4 = 3$

With obs, $v(\text{I},\text{II}) = \frac{1}{2}\cdot 4 + \frac{1}{2}\cdot 6 = 5$

Value of obs: = $\underline{\underline{2}}$

III	6	0	6
IV	0	6	6

$v(\text{III}) = \frac{1}{2}\cdot 4 + \frac{1}{2}\cdot 6 = 5$

$v(\text{IV}) = 4$

With obs: 6

Value of obs: 1

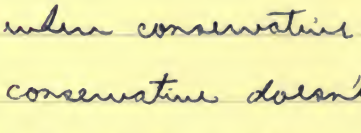
"Optimist" puts value of problem <u>without</u> info as higher than conservative (or Bayesian); ~~and puts value of info~~ but since there is a ceiling on ~~E~~ value of problem <u>with</u> info, he may value info less than either conservative or Bayesian if ceiling is reached; he will value ~~E~~ info more than conservative if ceiling is not reached.

An case:

	30 R	60 Y	B
~~III~~	~~6~~	~~0~~	~~6~~
~~IV~~	~~0~~	~~6~~	~~6~~

	30 R	60 Y	B
II	0	6	0
III	6	0	6

where conservative values info more than Bayesian (3 vs. 2), conservative doesn't know whether Y is more or less likely than B, but he know they are <u>exclusive</u>. Consider man who doesn't know whether ~~expected value~~ payoff of III is 0 or 6 and doesn't know whether ~~expectation~~ payoff of II is 0 or 6 but he does <u>know</u> their <u>correlation of outcome</u>; when payoff to II is 6, payoff to III is 0, and vice versa; hence an observation, which



will tell him which state applies [(Y) or (R, B)] will guarantee him a payoff of 6. Hyp: parallel paths pays more to conservative (than to Bayesian) if he knows outcomes are negatively correlated.

Regrets measure the value of various possible messages about true state of the world, relative to a given action. There is <u>no</u> reason to minimize the max of these.

Lit: Cowles ~~Co~~ Foundation Disc. Paper